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Tax Collection and Corruption in Fiscal Bodies

Alexander Vasin
Elena Panova

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CONTENTS

NON-TECHNICAL SUMMARY

The creation of an effective taxation system appears to be one of the most challenging problems for economies in transition. Budget-financed activities, such as social welfare, health care, fundamental science, *etc*., essentially depend on the existence of a carefully-designed taxation system. Unfortunately, tax evasion is widespread in Russia. The State Tax Service of Russia, through its collection and auditing efforts, makes an essential contribution to raising tax revenue. However, there is some statistical evidence that there exists significant potential to increase tax revenues, in particular by creating a more efficient taxation system. Some experts estimate that the "grey economy", in which no taxes are paid, amounts to 40% of Russian GDP.

Another serious problem is caused by the extremely low salary of tax inspectors. Receiving about $100 per month, some inspectors would seem to have strong incentives to engage in corrupt activities. The official statistics do not show any significant corruption in fiscal bodies, but there does exist a widespread opinion that the level of corruption within fiscal agencies is relatively substantial.

The present report considers several game-theory models related to tax evasion and the organization of audits for individual taxpayers and small enterprises. The first type of models studies the interaction between the tax authority and a group of taxpayers whose income is random, without taking into account the possibility of corruption. It is assumed that, at the end of the accounting period, each taxpayer declares her/his income to the tax inspectorate. The reported income is taxed according to the given tax rates, although a taxpayer may try to evade taxation by under-reporting. Recognising this possibility, the tax inspectorate audits some taxpayers. A detected tax evader is made to pay the evaded tax as well as a fine. Further, it is assumed that auditing is costly and that the central authority is interested in maximising net tax revenue (all the money received from taxes and fines minus audit expenditures) given the prevailing tax rates, fines and the costs of auditing. In the case of a homogeneous group of taxpayers, the only taxpayer-specific information available to the tax authority is declared incomes. Thus, the authority bases its choice of the probability of auditing on reported incomes. The purpose of this model is to find the optimal auditing rule given the tax rates and the income distribution.

The second part of the study is devoted to a model that takes the possibility of corruption into account. Tax inspectors are considered as another group of strategic players, in addition to the central tax authority and the taxpayers. There are two possible levels of income. The model assumes that a tax inspector who has discovered an instance of tax evasion may bargain with the detected evader over the amount of a bribe given in exchange for not revealing the evasion. In order to prevent this kind of corruption, the authority analyses the reports of inspectors, chooses to review some of the audits and penalises those inspectors who have not reported what the authority determined, in the review, to be instances of tax evasion. Thus, the authority's problem is to choose the frequencies with which to audit, at two levels — the audit of taxpayers by inspectors and the review of audits by the centre — depending on the information received. The first purpose of the analysis is to determine the optimal strategy for a tax authority that maximises net tax revenue given the tax rates, the penalties for evasion and for improper auditing, and the costs of audit and review. Another purpose is to provide a comparative statics analysis of net tax revenues with respect to the fine and tax rates both under optimal, as well as under constant non-optimal, probabilities of audit and review.

Our main practical conclusions are as follows.

1. The existing practical method of audit organisation is similar to the deterministic cut-off rule: taxpayers should be classified into groups with the same distribution of random income, and then every taxpayer with declared income less than some threshold specific for each group is audited. However, our analysis shows that this approach is not effective and should be replaced by the probabilistic cut-off rule: suspicious taxpayers should be audited with such minimal probability that makes tax evasion unprofitable. For any proportional tax and fine system, and for risk-neutral taxpayers, this probability is determined by the ratio of the tax and fine rates. Under the current rates of profit tax in Russia, this probability is about 0.15. Employment of a probabilistic cut-off rule either reduces the number of necessary audits or increases the set of enterprises that may be audited. This method may essentially increase net tax revenue.

2. The auditing strategy needs adjustment for corrupt inspectors in order to avoid the bribery of inspectors by taxpayers. We study two possibilities for such modification. One is to review the results of all audits and to penalise inspectors if the review reveals that the inspector has concealed tax evasion. The other way is to increase the auditing probability to such an extent that evasion is unprofitable, even if it is possible to bribe an inspector. Which method is less expensive depends on the relationship between different parameters. For instance, if the fine for evasion is sufficiently high, or a typical

bribe is close to the maximum value acceptable for an evader, then the second variant looks more preferable. If the fine is relatively small, or the bribe is close to the minimum acceptable to an inspector, then the first variant is favourable.

3. It is necessary to adjust the auditing strategy (the probabilities of audit and review) for every change in the tax rate or the penalty for evasion. Otherwise, an increase in these values may create new incentives for tax evasion or dishonest auditing and imply a decline in tax revenue.

1. INTRODUCTION

The creation of an effective taxation system appears to be one of the most challenging problems for economies in transition and particularly for Russia where tax evasion is widespread. Some experts estimate the "grey economy", in which no taxes are paid, as amounting to 40% of Russian GDP.

Corruption among tax inspectors represents another serious problem. The official data do not show any significant corruption in fiscal bodies (in particular, in 1995 there were 100 criminal investigations into bribery and only 6 inspectors were brought to trial), but there exists a widespread perception that the level of corruption within fiscal bodies is relatively substantial (see, for example, the interview of Hakamada in "Delovie lyudi", No.2, 1998). One possible factor that contributes to corruption among tax inspectors may be their relatively low salaries. Receiving about $100 per month, some inspectors would seem to have strong incentives to become corrupt.

The present paper develops several game-theory models related to tax evasion and corruption in the tax inspectorate. The first type of models studies the interaction between the tax authority and a group of taxpayers whose income is random, without taking into account the possibility of corruption. It is assumed that, at the end of the accounting period, each taxpayer declares his/her income to the tax inspectors. The reported income is taxed according to the given tax rates. However, a taxpayer may try to hide some part of income by under-reporting. If the taxpayer is audited, the inspector will inevitably uncover the true level of income. The detected tax evader is fined and made to pay the evaded tax. Further, it is assumed that auditing is costly and that the central authority is interested in maximising net tax revenue (*i.e.* the sum of taxes and penalties minus expenditures on audits) given the tax rates, fines and the costs of auditing. In the case of a homogeneous group of taxpayers, the only taxpayer-specific information available to the tax authority is the declared incomes. Thus, the authority must determine the probability of audit, using these declarations. The purpose of this model is to find the optimal auditing rule given the tax rates and income distribution. Section 3.1.1 discusses the solution to this problem with risk-neutral taxpayers when the fine for evasion is proportional to the evaded tax. Section 3.1.2 considers a similar problem with a progressive tax system and a fine for evasion proportional to the hidden income.

The second part of the study is devoted to a model that takes the possibility of corruption into account. Tax inspectors are modelled as another group of strategic players, in addition to the central tax authority and the taxpayers. Taxpayers can have two possible levels of income. The model assumes that a tax inspector who has discovered an instance of tax evasion may bargain with the detected evader over the size of a bribe given in exchange for not revealing the evasion. In order to prevent this kind of corruption, the authority analyses the reports of inspectors, chooses to review some of the inspector's audits and penalises those inspectors who have not reported tax evasion. Thus, the authority's problem is to choose the frequencies of both levels of audit — the audit of taxpayers by inspectors and the review of audits from the centre — depending on the information received.

The first purpose of the analysis is to determine the optimal strategy for the authority which maximises net tax revenue given the tax rates, the penalties for evasion and poor auditing, and the costs of audit and review. Another goal is to determine the comparative statics of net tax revenue with respect to financial penalties and tax rates, both under optimal audit and review conditions, and under given non-optimal probabilities. Section 3.2 solves this problem and shows that net tax revenue never decreases in the specified rates under the optimal rules, while it may decrease with fixed probabilities of auditing and reviewing.

Finally, Section 4 contains some implications of the results and discusses their application to the Russian economy. Proofs of Theorems are provided in an Appendix.

2. REVIEW OF LITERATURE

Tax evasion is a problem even in countries with developed tax systems. It has been analysed extensively within the economics literature. The connection between tax evasion and corruption has also been examined in a number of articles (see, for example, (Chander, Wilde, 1992; Соколовский, 1989; Васин, Агапова, 1993b)).

Several studies (Srinivasan, 1973; Cowell, Gordon, 1995) consider total tax revenue in situations where evasion is possible, but without taking corruption into account. A basic result on the optimal auditing of direct taxes appears in (Sanchez, Sobel, 1993). They consider a population of taxpayers with a random income distribution, characterised by a positive density over the given interval $[l, h]$. For any income I, the required tax is determined by a tax schedule $T(I)$ established by the government. The tax strictly increases in I. The interaction between taxpayers and the tax administration follows the same procedure as described in the above introduction. The administration sets a probability $p(I)$ of auditing, depending on reported income I.

If an audit reveals under-reporting, the penalty imposed on the taxpayer is proportional to the unpaid tax with the coefficient of proportionality $1 + \pi > 1$ (since the penalty includes the unpaid tax).

If the tax administration aims to maximise the revenue from taxes and fines net of auditing costs, Sanchez and Sobel (1993) show that the optimal auditing policy always belongs to the class of cut-off rules:

$$p^*(I) = \begin{cases} \dfrac{1}{1+\pi}, & I < \bar{I}, \\ 0, & I \geq \bar{I} \end{cases}$$

for some $\bar{I} \in [I, h]$. Thus, every reported income $I < I$ is audited with probability $1/(1+\pi)$, which is the minimum probability which makes it unprofitable to report I for any taxpayer with income $I' > I$. Section 3.1 below generalises this result by considering arbitrary distributions of income.

Cowell and Gordon (1995) compare different audit strategies available to a tax authority that collects indirect taxes. The authors model tax evasion as follows: taxpayers choose between taxable activities on the regular market and unreported activities on an informal market. Individuals are audited and, where they are found to be undertaking irregular activities, are fined and made to repay the evaded tax. One possible strategy is to audit randomly, with some fixed probability that any taxpayer is investigated.

An alternative policy is to take into account what the authority knows about each taxpayer. Cowell and Gordon study a simple form of this approach where the authority conditions the probability of audit on reported turnover via a cut-off rule: those reporting less (no less) than a certain amount are always (never) audited. Cowell and Gordon establish conditions under which the optimal random audit is better than the optimal cut-off rule, and vice versa. However, as Siniscalco notes in his discussion of Cowell and Gordon's model (Cowell and Gordon, 1995, p.197), the optimal audit strategy in general does not belong to any of the specified classes.

Chander and Wilde (1992), in "Corruption in tax administration" (hereafter the CW model), focus on the interaction between taxpayers and tax inspectors (*i.e.* auditors), taking into account the possibility of corruption. According to this model, a taxpayer can have either high or low income with some probability and is expected to pay taxes corresponding to actual income. Nevertheless, a taxpayer who earns high income may under-report, claiming low income. If that taxpayer is audited, any tax evasion is detected, with certainty, by the tax inspector. It is possible, however, that the inspector can be bribed to conceal the results of the audit. In this case, the taxpayer is free from both taxation (of high income) and the penalty for tax evasion, but there exists a chance that the inspector and evader will be caught, in which case both suffer additional costs.

The players (*i.e*. taxpayers and auditors) are assumed to be risk neutral. Taxpayers minimise their expected total expenditures on taxes, bribes and the penalties (for tax evasion and bribery) applied when their non-compliance is detected. Tax inspectors maximise their expected net gains (that is, their expected income from bribes minus penalties for exposed bribe-taking). A strategy of a taxpayer includes a probability of engaging in tax evasion and the decision of whether or not to bribe the tax inspector if evasion is detected. A strategy of an inspector concerns whether or not to take a bribe. A strategy of reviewing audits by the central tax agency is chosen to maximise net tax revenue. The strategies of all agents are assumed to form a Nash equilibrium. Under these assumptions, the paper studies comparative statics and obtains complex and ambiguous results: sometimes an increase in the tax rate may decrease net tax revenues.

In the present report, we investigate comparative statics with respect to the rates of financial penalties. There are several important differences between the CW analysis and our approach. The CW model assumes that the probability of auditing is determined by the Nash equilibrium strategies of the players, while the probability of bribery exposure is fixed. However, both these values in practice may be controlled by the central authority.

In common with Cowell and Gordon, we assume that the strategy of the tax authority (including the rules for audit and review and the penalties for violations) is common knowledge. The behaviour of taxpayers and inspectors is rational in the following sense. Everyone is an expected utility-maximiser, risk-neutral with respect to income after tax and fines. We assume that, if the minimum bribe value acceptable for an inspector is less than the maximum size of a bribe that is acceptable to a taxpayer, then they come to an agreement that shares the surplus in some proportion. As the acceptable sizes of bribes depend on tax and penalty rates, taxpayers and inspectors determine their strategies depending on the parameters chosen by the central authority.

So, in common with Cowell and Gordon, and in contrast to the CW model, we determine the strategy of the tax authority within a principal-agent framework. Another difference with the CW model is that we consider optimal auditing rules (as opposed to fixed audit probabilities) when studying the comparative statics of net tax revenue with respect to the fine and tax rates. This seems to be reasonable since the auditing rules are more flexible than the rates established by the legislature.

Section 3.2 shows that, under our approach, we obtain clear and tractable results, avoiding some of the ambiguous findings of the CW model.

Obviously, the maximisation of tax revenue is not the only goal of economic regulation. Optimal income redistribution through taxation presents a related problem that is widely discussed in the literature (see Piketti, 1992; Мовшович, Богданова, Крупенина, 1997, *etc*.). We do not consider the redistribution motive for taxation in our model. Note, however, that the optimal tax rates that emerge from a more general maximisation problem may be viewed as defining the desirable tax rates that are the exogenous parameters of our model.

3. THE MODELS

3.1. A model without corruption

Consider a homogeneous group of taxpayers with distribution of income I given by density $\nu(I)$, $I \geq 0$. Let $T(I)$ denote the legal tax liability for income I. The behaviour of taxpayers is characterised by the function $I_d(I)$, which determines the declared value of income I_d depending on actual income I. The tax authority establishes the probability $p(I_d)$ of audit for those who declare income I_d. An audit always reveals the taxpayer's true income. The fine for cheating is given by the function $F(I, I_d)$, and includes the unpaid tax.

The optimal strategy of a taxpayer for any given $p(\cdot)$ is determined by the solution to the problem:

$$I_d(I, p(\cdot)) \to \max \{(I - T(I_d) - p(\mathrm{I_d}))F(I, I_d)\},\ I_d \in [0, I].$$

Let c denote the cost of one audit. Then, for any $p(\cdot)$, the net tax revenue of the central authority is:

$$R(p(\cdot)) = \int \{T(I_d(I, p(\cdot)) + p(I_d(I, p(\cdot))\,(F(I, I_d(I, p(\cdot))) - c\}\, d\nu(I)\ .$$

and the authority's problem is to find the strategy $p^{*}(\cdot)$ which maximises this value.

3.1.1. The case of a proportional tax and fine structure. Let

$T(I_d) = t\, I_d, \quad F(I, I_d) = (f + t)(I - I_d),$

where t and f are positive scalars. The following proposition determines the minimum probability of being audited which guarantees that a taxpayer with income I will declare the true level of income.

Proposition 1. $I_d(I, p(\cdot)) = I$ if $p(I_d) \geq t/(f + t)$ for any $I_d < I$.

Thus, if a taxpayer's income is *a priori* unbounded, then the optimal audit strategy among those which reveal the true income of every taxpayer is a random audit with probability

$$\hat{p} \equiv \frac{t}{t+f}.$$

However, this strategy is generally sub-optimal if income is *a priori* bounded.

For any $\bar{I}$ consider the following cut-off rule: $p(I, \bar{I}) = \hat{p}$ if $I < \bar{I}$, and $p(I, \bar{I}) = 0$ otherwise. Under this rule, any taxpayer with income $I < \bar{I}$ declares I, and everyone with income $I > \bar{I}$ declares $\bar{I}$. Note that this rule is clearly optimal if all taxpayers have income $\bar{I}$. There exist many other rules corresponding to different non-increasing probabilities $p(I_d)$, which we cannot *a priori* reject as sub-optimal. The following Theorem shows that the optimal rule, however, always belongs to the class of cut-off rules (including the random rule $\hat{p}$ as the limiting case). Let $dv / dI = \rho$ be the density of income.

Theorem. The strategy $p(I_d) = t/(t + f)$ for any I_d is optimal if

$$\int_{I \geq \bar{I}} (t(I - \bar{I}) - \frac{ct}{f+t}) \rho(I)\, dI \geq 0 \quad \text{for any } \bar{I}; \tag{1}$$

otherwise, the cut-off rule with some $\bar{I}$ such that (1) fails is the optimal one. Proof of this Theorem and sequenced propositions are given in the Appendix.

Let us discuss condition (1). This condition is met with equality if $\rho(I) = k \cdot \exp(-I(t + f)/c)$, and it holds as an inequality if $|\rho'/\rho| < (f + t)/c$ for $I > \bar{I}$ (that is, for distributions of income with a "heavy tail").

If (1) fails to hold, then the problem is: to find $\bar{I}$ that maximises $R(p(\cdot\,|\,\bar{I})) \equiv^{\text{def}} R(\bar{I})$. This function is multi-extreme in the general case. However, for a wide class of distributions the optimal value is unique.

Proposition 2. Let ρ have a unique maximum I_M, and let $|\rho'/\rho|$ increase in I when $I \in [I_M, \hat{I}]$ and decrease in I when $I > \hat{I}$. Then $R(\bar{I})$ has, at most, two local maxima including $+\infty$.

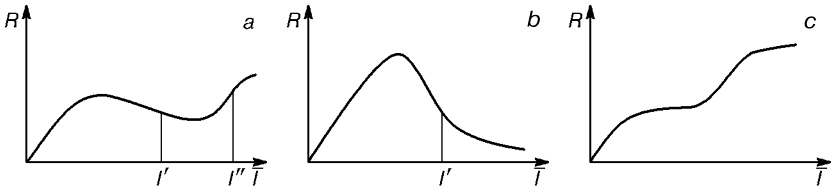

Figure. Different variants of the relationship between R and $\bar{I}$: *a* — two maxima; *b, c* — unique maximum.

Note that the lognormal distribution with

$$\rho(I) = \frac{1}{I\sigma\sqrt{2\pi}} \exp\left(\frac{-\ln^2(I/\bar{I})}{2\sigma^2}\right)$$

meets the conditions of Proposition 2 (in this case $\ln I_M = \ln \bar{I} - \sigma^2$, $\ln \hat{I} = \ln \bar{I} + \sigma^2$, and two optima are possible).

Now, let us show that any increasing continuous tax $T(I)$ and any penalty proportional to the unpaid tax ($F(I, I_d) = (1 + \pi)(T(I) - T(I_d))$ is equivalent to the linear case that has already been examined. Indeed, let $dT(I)/dI_+ = t(I) > 0$. We can rewrite the problems for a taxpayer and the authority as follows:

$$T_d(T) \to \min\{T_d + p(T_d)(1+\pi)(T-T_d)\},$$

$$p(\cdot) \to \max \int \{T_d(T) + p(T_d(T))[(1+\pi)(T-T_d) - c]\}\, d\,\bar{\mu}(T),$$

where $\bar{\mu}(T) = \nu(I(T))$ for $I(T)$ such that $T(I(T)) = T$. If $\nu(I)$ is differentiable at $I(T)$ then

$$\frac{d\bar{\mu}(T)}{dT} = \frac{1}{t(I)} \frac{d\nu(I(T))}{dI}.$$

3.1.2. Progressive tax and fine linearly depending on non-declared income. Now, let us assume that the tax is progressive and that the fine linearly depends on non-declared income. (This is currently the case in Russian legislation.) That is, let $T(I)$ be a monotone convex function such that $T(0) = 0$, $dT/dI_+ =^{def} t(I)$ does not decrease and equals to t_{max} for sufficiently large I, and $F(I, I_d) = T(I) - T(I_d) + f(I - I_d)$. The counterpart of Proposition 1 in this case is as follows.

Proposition 3. $I_d(I)=I$ if $p(I_d)\geq \hat{p}(I_d, I)=^{def}(T(I)-T(I_d))/(T(I)-T(I_d)+f(I-I_d))$ for any $I_d < I$.

Proceeding from this result, the optimal audit strategy among those which reveal the true income of every taxpayer is random auditing with a probability $\hat{p} = t_{max}/(t_{max} + f)$. However, the "competitors" of this strategy are more complicated than the cut-off rules considered for a proportional tax. For a progressive tax, any rule $p(I_d, \bar{I})$ with $t = t_{max}$ is dominated by the following audit strategy:

$$\bar{p}(I_d \mid \bar{I}) = \hat{p}(I_d, \bar{I})\text{, if } I_d < \bar{I}\text{; else } \bar{p}(I_d \mid \bar{I}) = 0 .$$

That is, lower declared incomes can be audited with probability less than $\hat{p}$. Moreover, sometimes the strategy $\bar{p}(\cdot \mid \bar{I})$ is dominated by another:

$$\tilde{p}(I_d \mid \bar{I}) = \hat{p}(I_d, \bar{I})\text{, if } I_d < \bar{I}\text{, else } \tilde{p}(I_d \mid \bar{I}) = \hat{p}(0, \bar{I})\text{,}$$

which differs from the previous strategy where high incomes are audited with the same probability as an income declaration of 0.

Conjecture. The optimal audit rule for progressive tax is either $\bar{p}(\cdot \mid \bar{I})$ or $\tilde{p}(\cdot \mid \bar{I})$ for some $\bar{I}$, $0 \leq \bar{I} \leq \infty$.

If the conjecture is true, then the problem of the optimal audit strategy search is reduced to the following: to find $\bar{I}$ which maximises

$$R(\bar{I}) = \int_0^{\bar{I}} \left(T(I) - c\hat{p}(I_d, \bar{I})\right)\rho(I)\,dI + \\ + \max\left\{T(\bar{I})\int_{\bar{I}} \rho(I)\,dI,\ \hat{p}(0, \bar{I})\int_{\bar{I}} \left(T(I) + fI - c\right)\rho(I)\,dI\right\}. \tag{2}$$

In practice, it is nearly impossible to evaluate the income distribution *a priori* for some group of taxpayers. This analysis suggests that a reasonable approach under such conditions is to set $p(I_d)$ slightly more than t/f for any I_d. This will force risk neutral or risk averse taxpayers to declare their true incomes and enables the authority to improve its information about income distribution, and eventually to introduce an improved auditing rule if it turns out to be unprofitable to audit taxpayers who declare relatively high incomes. Of course, for some groups this approach will be sufficiently unpromising, perhaps because of high auditing costs, such that they should be exempt from the audit process from the beginning. But this is a different issue.

3.2. A model with corrupt auditors

This section considers a model that takes into account the possibility that a detected tax evader will bribe an auditor. Assume that there are only two possible levels of income $I_L < I_H$, obtained with probabilities $1 - q$ and q, respectively. The low-income level is free of tax and the tax on high income is T. Thus, taxpayers with true income I_H may have an incentive to report I_L. If a taxpayer reports low income, the tax agency may wish to conduct an audit. An audit costs c, and always reveals the true income. The fine for under-reporting is F, and includes the original tax liability. An auditor may be bribed, however, inducing him or her to suppress the result of the audit, thus shielding a taxpayer who reports I_L instead of actual income I_H from a fine for under-reporting.

The central authority sometimes checks auditors who confirm low incomes and penalises them if the review reveals that the inspector has concealed tax evasion. (They are penalised for poor auditing, not for accepting bribes, because of the difficulty of proving that a bribe was accepted.) The probabilities p and p_c of auditing by the inspector and reviewing by the central tax authority, respectively, are themselves established by the authority. The penalty for a poor audit is equivalent to a monetary fine of value $\tilde{F}$, but it is assumed that only some share $\delta \in (0,1)$ of this value represents revenue to the budget. The other component of the fine borne by the auditor could be considered to be a loss of future income due to discharge or a loss of reputation. (Note that $\delta = 0$ in the CW model, *i.e.* none of the fine becomes revenue to the budget.) The cost of reviewing an audit is $\tilde{c}$. The central authority aims to maximise net tax revenue, including taxes and its share of fines minus all audit costs.

Our purpose is to find the optimal probabilities p and p_c and then to study comparative statics with respect to the size of the fine. We start by discussing the bargaining over the amount of the bribe, b, in the case where a tax evader has been caught by an auditor. Bribing is profitable for the taxpayer and the auditor if, respectively, $b + p_c F < F$ and $b > p_c \tilde{F}$. Thus, bribing is possible if:

$$F(1 - p_c) > p_c \tilde{F} . \tag{3}$$

If this inequality holds, let $b = \gamma F(1 - p_c) + (1 - \gamma) p_c \tilde{F}$, $\gamma \in (0,1)$.

A taxpayer with high income will cheat if $p(b + p_c F) < T$. If (3) does not hold, then he or she will cheat, but not bribe, if $pF < T$.

Thus, we should consider the following cases.

a) $F(1 - p_c) > p_c \tilde{F}$, $p(b + p_c F) < T$. Subsequently, taxpayers evade, auditors take bribes, and the net tax revenue per taxpayer is:

$$R = p\left(p_c (q(F + \tilde{F}\delta) - \tilde{c}) - c\right).$$

b) $F(1 - p_c) < p_c F$, $pF < T$. In this case, taxpayers evade but there is no bribery and:

$$R = p\left(qF - c - (1 - q) p_c \tilde{c}\right).$$

c.1) $pF > T$ and $F(1 - p_c) < p_c \tilde{F}$, or

c.2) $p(b + p_c F) > T$ and $F(1 - p_c) > p_c \tilde{F}$. Under these conditions, there is no tax evasion, and

$$R = qT - (1 - q)p(c + p_c \tilde{c}).$$

Denote $\hat{p} = T / F$, $\hat{p}_c = F / (F + \tilde{F})$.

Proposition 4. Under condition a), revenue R tends to the *supremum* R_a when p tends to $\hat{p}$ and p_c tends to $\hat{p}_c$. The *supremum* R_b under condition b) and maximum R_{c1} under c.1), moreover, correspond to the same probabilities, $R_a < R_b < R_{c1}$. Under condition c.2), the optimal probabilities are the same and maximal revenue is

$$R_{c2} = R_{c1} = qT - \hat{p}(c + \hat{p}_c \tilde{c}),$$

if

$$\hat{p}_c < c \frac{1 - \gamma}{\tilde{c}\gamma} ; \tag{4}$$

otherwise the optimal strategy is $p = T/\gamma F$, $p_c = 0$ and maximal revenue is

$$\overline{R}_{c2} = qT - cT\frac{1-q}{\gamma F}.$$

Note 1. Under $\delta = 1$ and the optimal probabilities, taxpayers pay the same amount under conditions a), b) and c.1). However, the costs of inspection fall dramatically when the system passes from a) to b) or from b) to c.1).

Note 2. Inequality (4) holds, in particular, if γ tends to 0, that is, where taxpayers have bargaining power over the amount of the bribe. If γ tends to 1 (that is, the inspectors have the power), then the optimal strategy is not to review the inspectors but to set a higher probability of being audited.

Thus, the net tax revenue under the optimal auditing strategy is

$$R^* = T\left\{q - (1-q)\min\left[\frac{c}{F} + \frac{\tilde{c}}{F+\tilde{F}},\ \frac{c}{\gamma F}\right]\right\}.$$

Proceeding from this relationship, the comparative statics of R with respect to the fines and the tax is quite clear: R increases in T and F, it also increases in $\tilde{F}$ while

$$\frac{\tilde{F}}{F} \geq \frac{\tilde{c}}{c(1/\gamma)^{-1} - 1};$$

otherwise, R does not depend on $\tilde{F}$. (Of course, these comparative statics hold only under a fixed tax base; see the next section for some development of this point).

However, if p and p_c are fixed then the comparative statics is more complicated. Within each area a), b), c.1) and c.2), the revenue is monotone in the fines and the amount of the tax (in accordance with common sense), but the transitions can bring surprises. Consider two examples.

1. Let $p < \hat{p}$, $p_c = \tilde{p}_c$. Consider a small increase in the fine: $F' = F + dF$. Then, the system shifts from area b) to area a), and the revenue R suffers a sharp fall.
2. Let $p = \hat{p}$, $p_c = \tilde{p}_c$. Here, a small increase in the tax implies a shift from area (c.1.) to area (b) and a sharp fall in R.

4. CONCLUDING REMARKS AND APPLICATIONS TO THE RUSSIAN ECONOMY

The results of section 3 enable us to propose the following approach to improving the organisation of the tax inspectorate.

1. Proceeding from *a priori* information, taxpayers should be classified into groups with the same distribution concerning random income. For instance, for any enterprise, its account balance, together with data from the banks where the taxpayer has accounts, would enable an estimate of the maximum and minimum possible income from operations on the legal market. A distribution of this interval can be evaluated by using statistical data on the probabilities of cheating for every kind of cost or exemption which the taxpayer declares.
2. For each sub-group of taxpayers with the same distribution of income, an optimal auditing strategy should be calculated. According to Section 3.1, this strategy is a modified cut-off rule $\overline{p}(I_d \mid \bar{I})$ defined in 3.1.2. In particular, for a proportional tax rate t and fine rate f, taxpayers who declared income I_d less then $\bar{I}$, should be audited with probability $t/(t+f)$.
3. The auditing strategy needs an adjustment to account for corrupt inspectors in order to avoid the bribery of inspectors by taxpayers. Section 3.2 considers two possibilities for such a modification. One is to review the results of audits with a certain degree of probability and to penalise inspectors if the review reveals that the inspector has concealed tax evasion. The other way is to increase the auditing probability to such an extent that evasion is unprofitable, even if it is possible to bribe an inspector. Which method is less expensive depends on the relations between the parameters of the model. For instance, if the fine for evasion is sufficiently high, or a typical bribe is close to the maximum value acceptable for an evader, then the second approach looks preferable. If the fine is relatively small, or the bribe is close to the minimum acceptable for an inspector, then the first approach is better.
4. It is necessary to adjust the auditing strategy (the probabilities of audit and review) for every change in the tax rate or the penalty for evasion. Otherwise, an increase in these values may create new incentives for tax evasion or dishonest auditing and imply a decline in tax revenue.
5. Last but not least, the previous results are valid for one short interaction between taxpayers and inspectors who match in random. It is

necessary to avoid long-term relationships between taxpayers and inspectors. Otherwise, each pair has a strong motivation for "co-operative" behaviour which minimises tax revenue (see the standard results on repeated games and their discussion by Tirole (1992).

Let us discuss some of these conclusions applied to the collection of the profit tax in Russia. (In 1996, the share of the profit tax in the revenue of the state budget was 18%.) According to Russian legislation (see *Налоги*, 1997), the profit tax rate is t = 0.35, and the fine rate is f = 2, that is, the optimal probability of being audited is $\hat{p} = 7/47 \approx 0.15$.

Section 3.1 shows that the cut-off rule is optimal in this case. That is, enterprises which declare income (profit) less than $\bar{I}$ should be audited with probability $t/(t + f)$.

In order to choose the optimal value of threshold $\bar{I}$, it is necessary to divide enterprises into homogeneous groups and to evaluate the income distribution within each one. Proceeding from the current situation of mass tax evasion, we may expect that, at the first stage, it will be optimal to audit almost all enterprises. According to Theorem, the maximum possible income for a given group of enterprises relates to $\bar{I}$ as follows: $I_{\max} - \bar{I} > c/(t+f) \approx 0{,}43c$.

The cost of auditing essentially depends on the method of evasion. Let us roughly evaluate this cost. Evaluation by audit experts indicates that, for a firm with a turnover of $30,000 and a profit of $15,000 per quarter, a document audit usually lasts 2 – 3 days. However, the method of converting profit to cash through a fictitious firm — an evasion technique widely used in Russia — requires additional expenses in searching and auditing such firms. Let us take one month as the upper value of the total length of the audit. The salary of inspectors and tax policemen was about $100 – 500 per month prior to August 1998, so let c = $300. We can rewrite our evaluation as follows:

$$\frac{I_{\max} - \bar{I}}{I_{\max}} > 0.43 \frac{c}{I_{\max}},$$

thus it would seem beneficial to audit an enterprise if the declared profit differs from the *a priori* upper value by more than 1%.

Currently in Russia there are about 20 firms per inspector, so under specified $\hat{p}$ he should audit an average of three enterprises in one quarter, which seems to be feasible.

Note 3. In the present work, we do not consider the dynamics of taxpayers' behaviour under a changing audit strategy. A gradual decrease in the share of taxpayers who evade taxes may be expected when $p(I_d) > \hat{p}$ for any $I_d < I$. Thus, tax revenues will increase in the share of audited enterprises if the current share of tax evaders with hidden incomes is sufficiently large. More precisely,

$$R(I_d) \stackrel{\text{def}}{=} \int \pi(I \mid I_d)\left[(t+f)(I-I_d)-c\right]dI > 0,$$

where $\pi(I \mid I_d)$ is the density of income distribution for taxpayers who declared I_d. In order to increase the current revenue, it is reasonable to evaluate $\pi(I \mid I_d)$ on the basis of conducted audits and to increase the share of audited enterprises for such I_d where $R(I_d) > 0$, by beginning with maximum values and continuing until the resources of the inspectorate are spent. (We may expect that the organisation of additional audits will become more expensive and auditing should be discontinued when the value of $R(I_d)$ becomes 0.) Of course, we should not decrease the probability of audit for other taxpayers with $I_d < \bar{I}$.

The existing method of the analysis of the financial and production activity of enterprises and organisations (*Методика проведения анализа финансово-хозяйственной деятельности предприятий и организаций*, 1997) worked out for tax inspectorates provides a good basis for solving the problem of the selection of enterprises for audit. According to this method, enterprises are ranked according to the difference between declared income (and some other parameters) and typical values calculated from the past activities of similar enterprises. It is proposed to audit enterprises where this difference exceeds some threshold — that is, similar to the deterministic cut-off rule (see Cowell, Gordon, 1995). However, we have shown above that such a rule is not effective. Employment of the probabilistic cut-off rule makes the set of enterprises which may be audited substantially wider and reduces the probability of tax evasion.

In our discussion of the implementation problems, we should take into account the possible irrelevance of our assumptions to the actual conditions in some regions where corruption is organised, that is, where bribes collected instead of taxes go to the top officials in the local administrations. Under such conditions, an inspector who does not take bribes will not get a job and the measures discussed above will not be effective. Another important note: the tax legislation should be changed before the reorganisation of tax inspection, because honest behaviour under the current level of taxes would lead the majority of enterprises into bankruptcy. One more potentially dangerous factor, which is not reflected in our models, is the competition between regions for taxpayers, leading to the existence of several "offshore zones" on the territory of Russia.

Thus, much work must be done on the complex improvement of the Russian tax system. We hope that our results and the approach developed in the present paper will be useful for the continuation of this work.

APPENDICES

A. Proof of Theorem

If (1) fails for some $\bar{I}$ then $R(\hat{p}) < R(p(\cdot), \bar{I})$ since the left hand side of (1) is the difference between these values. Now, for any $I_1 < I_2 < \ldots < I_k$ and $\hat{p} \geq p_1 > p_2 > \ldots > p_k \geq 0$, consider the k-level strategy $p(\cdot)$ such that $p(I) = \hat{p}$ if $I < I_1$, $p(I) = p_l$ if $I_l \leq I < I_{l+1}$, $l = 2, \ldots, k$. The behaviour of a taxpayer under such a strategy is as follows. For $l = 1, \ldots, k-1$, $\bar{I}_l$ determines the income value, such that the expected income after taxes and fines is the same whether a taxpayer declares I_l or I_{l+1}. That is,

$$tI_l + p_l(t+f)(\bar{I}_l - I_l) = tI_{l+1} + p_{l+1}(t-f)(\bar{I}_l - I_{l+1}) .$$

Then

$$\bar{I}_l = \frac{t}{t+f}\frac{I_{l+1} - I_l}{p_l - p_{l+1}} + \frac{p_l I_l - p_{l+1} I_{l+1}}{p_l - p_{l+1}} . \qquad \text{(A.1)}$$

Note that $\bar{I}_l > I_l$, and $\bar{I}_l$ is increasing in a monotone fashion in p_{l+1}. If p_l is fixed and p_{l+1} tends to p_l, then $\bar{I}_l$ tends to ∞.

Let $\bar{I}_1 < \bar{I}_2 < \ldots < \bar{I}_{k-1}$, $\bar{I}_k \stackrel{\text{def}}{=} \infty$. Thus, taxpayers with income $I \in [0, I_1]$ declare their true income, for $I \in [I_1, \bar{I}_1]$, $I_d(I) = I_1$, and for $I \in (\bar{I}_{l-1}, \bar{I}_l)$, $I_d(I) = I_l$, $l = 2, \ldots, k$. We assume that, if several values of declared income correspond to the same expected income after taxes and fines, then a taxpayer declares the value closest to actual income.

If $\bar{I}_l \leq \bar{I}_{l-1}$ for some l, then the taxpayer never declares I_l. There exists an auditing strategy, $m < k$, which implements the same behaviour from taxpayers and yields the same revenue. (Consider an auditing strategy which differs from the original only in that $p(I) = p_{l-1}$ for $I_l \leq I < I_{l+1}$.)

Let us prove by induction that any k-step auditing strategy is dominated by some 1-step strategy with $p_1 = 0$ (*i.e.*, by a new cut-off rule).

Consider a 2-step strategy $S(I_1, I_2, p_1, p_2)$. Then, $\bar{I}_1$ is given by (A.1). Let us determine a variation $dp = (d_1, d_2)$ such that $\bar{I}_1$ stays the same for any permissible strategy $S(I_1, I_2, p_1+xd_1, p_2+xd_2)$. Then,

$$d_1 = d_2 \frac{\bar{I}_1 - I_2}{\bar{I}_1 - I_1};$$

since

$$p_1 = p_2 \frac{\bar{I}_1 - I_2}{\bar{I}_1 - I_1} + \hat{p}\frac{I_2 - I_1}{\bar{I}_1 - I_1}.$$

Let $p(x)=p + xdp$, $d_2 = 1$. The maximum permissible value of x corresponds to $p_1(x_{max}) = p_2(x_{max}) = \hat{p}$, that is, to the random auditing rule. Under the minimum value,

$$\mathrm{p}_2(\mathrm{x}_{\mathrm{min}}) = 0, \quad \mathrm{p}_1(\mathrm{x}_{\mathrm{min}}) = \hat{p}\frac{I_2 - I_1}{\bar{I}_1 - I_1}.$$

The net revenue linearly depends on x:

$$\frac{dR(x)}{dx} = \frac{\bar{I}_1 - I_2}{\bar{I}_1 - I_1}\int_{I_1}^{\bar{I}_1}\big((t+f)(I - I_1) - c\big)d\nu + \int_{I_1}^{\infty}\big((t+f)(I - I_2) - c\big)d\nu. \tag{A.2}$$

If this value is non-negative then $S(I_1, I_2, p_1, p_2)$ is worse than the random auditing rule, otherwise this strategy is dominated by

$$S(I_1, I_2, \hat{p}\frac{I_2 - I_1}{\bar{I}_1 - I_1}, 0).$$

For the last strategy, the net revenue is equal to the convex combination:

$$\lambda R\big(S(I_1, 0)\big) + (1-\lambda)R\big(S(\bar{I}_1, 0)\big) \text{ with } \lambda = (\bar{I}_1 - I_2)/(\bar{I}_1 - I_1). \tag{A.3}$$

Thus, one of these cut-off rules is not worse than the original strategy.

Now, consider any k-step strategy, $S(I_1, \ldots, I_k, p_1, \ldots, p_k)$. If $\bar{I}_l \le \bar{I}_{l-1}$ for some l, then we can argue by intuition. If $p_k > 0$, then let us determine a variation $d\mathbf{p} = (d_1, \ldots, d_k)$ such that the values $\bar{I}_1, \ldots, \bar{I}_{k-1}$ are the same for any strategy $S(I_1, \ldots, I_k, \mathbf{p}(x))$, where $\mathbf{p}(x) = \mathbf{p} + xd\mathbf{p}$.

It suffices to set $d_k = 1$, and $d_{l-1} = d_l y_l$, where $y_l = (\bar{I}_{l-1} - I_l)/(\bar{I}_{l-1} - I_{l-1})$, $l = k, \ldots, 2$. The *supremum* x_{max} of permissible x corresponds to $p_l(x_{max}) = \hat{p}$ for any $l = 1, \ldots, k$, and the minimum value is determined by $p_k(x_{min}) = 0$. The subsequent argument is the same as in the case of $k = 2$.

Finally, note that any non-increasing function $p(\cdot)$ may be approximated by a k-level strategy with any degree of precision with respect to the value of R, if k is large enough.

B. Proof of Proposition 2

By definition of $R(\bar{l})$,

$$R^{'}(\bar{l}) = -\rho(\bar{l})\frac{ct}{t+f} + t\int_{\bar{l}} \rho(l)dl, \tag{B.1}$$

$$R^{''}(\bar{l}) = t\rho(\bar{l})\left\{1 + \frac{c}{t+f}\frac{\rho'}{\rho}\right\}. \tag{B.2}$$

The equation

$$\frac{\rho'}{\rho} = -\frac{t+f}{c} \tag{B.3}$$

does not have solutions when $l \le l_d$ and does not have more than one solution in $[l_M, \hat{l}]$. If such root $l^{'}$ exists then $R^{''}(l) \le 0$ when $\bar{l} \le l^{'}$ and one more root $l^{''}$ of this equation may exist in the interval (l_M, ∞). Then $R^{''}(l) \ge 0$ when $l^{'} \le \bar{l} \le l^{''}$ and $R^{''}(\bar{l}) \le 0$ when $\bar{l} > l^{''}$. If $R^{'}(l^{'}) < 0$, then one local maximum is situated in the interval $\bar{l} \le l^{'}$. If $R^{'}(l^{''}) > 0$, then the second local maximum is reached when $l \to \infty$ (because $R^{'}(\bar{l}) \to \infty$ when $\bar{l} \to \infty$). If the second root does not exist, then the first maximum is unique. In the case that $R^{'}(l^{'}) \ge 0$, the only possible maximum is $\bar{l} = \infty$.

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