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Does Lower Inflation Imply Lower Price Uncertainty?

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1. INTRODUCTION

In the literature considering the problems of inflation, one often comes upon a hypothesis that, under high inflation, it is more difficult to predict what will be the general price level in the future. For example, see Okun (1971, 1975). This association has been repeatedly pointed to by such influential economists as M. Friedman (1977). In fact, it is difficult to find a recent discussion of the consequences of inflation in which this link is not mentioned (Driffill *et al*., 1990; Paldam, 1994; O'Reilly, 1998; Sinn, 1999).

It is possible to carry out a thought experiment and imagine an economy in which there is no price uncertainty; thus it becomes clear that, if economic agents could precisely predict the price level, then the many negative consequences of price increases would not exist.[1]

Let us consider how price uncertainty influences the contracting process. If a sum of money appears in a contract then, at the moment when the contract is drawn up, the parties have to take into account the depreciation of this sum at the point of payment. Thus, they put expectations of future price level behaviour into the terms of the agreement. If these forecasts were incorrect and the prices turned out to be higher or lower than was expected then one party would receive more than it wanted and the other would receive less. It is clear from this that unexpected changes in the rate of inflation cause redistribution in the economy. The scope of this redistribution could be quite serious if the rate of inflation increases or decreases sharply and could cause undesirable social effects.

From the point of view of the general level of economic activity, it is not this redistribution itself that is important, but rather the risk associated with it. Economic agents anticipate this and reduce the economic activity which is subject to such risk in order to hedge against it; *i.e*. they do not contract at all or switch to shorter contracts.

The possibility to conclude long-term contracts is important for many sectors. For example, this is very important for the labour and real estate markets in which changing the terms and conditions and searching for new partners are associated with high costs. This is also true of

[1] However, such an economy would have other costs of inflation: deadweight loss due to depreciation of money balances, "menu costs", costs of tax distortions, *etc*.

long-term co-operation between firms and, probably the most important, of the investment market. Thus, price uncertainty directly bears upon factors determining the rates of economic growth.

An unexpected change in the price level results in an unexpected reduction (or an increase under disinflation) of real money balances. The price increases which followed the "August crisis" of 1998 in Russia could be cited as one such example. An event of this kind could be compared to unexpected money reforms. Similarly to money reforms, unexpected changes in the price level result in a redistribution of wealth.

The technical inconveniences of price uncertainty are connected with planning in nominal terms. Apparently, it is most convenient to plan future economic activity in terms of the same units which are used for payments in the economy. Uncertainty concerning the purchasing power of the unit, in terms of which planning is carried out, makes economic activity on the whole less regular.

Thus the importance of investigating the link between the rate of inflation and price uncertainty arises from the role which price uncertainty plays in decisions concerning long-term contracts and long-run planning in normal terms.

The problem could be summarised as follows. By means of inflation, the State collects an inflation tax from money base holdings. However, in doing so it makes money less suitable as a unit of account, since price uncertainty makes the purchasing power of money less predictable. Money is a universal measure in the economy. Money is used for stating contract terms, for negotiating prices, for economic analysis and planning. And the more predictable is its purchasing power, the better it plays this role.

The link between inflation and price uncertainty could explain why inflation is considered as an undesirable phenomenon, *i.e.* inflation can be bad not by itself but on account of the uncertainty in the future price level that it causes. If this is the case, then fighting inflation must mean fighting not with the growth of prices but rather with price growth instability. An unwise fight with inflation can be more harmful than inflation itself as far as it increases price variability and uncertainty.

In order to position this study in the context of the problem under discussion, we will name the questions which could be asked by a researcher:

- How can price uncertainty be measured?
- Is there at all a link between inflation and price uncertainty?

- If the link does exist, then what is the direction of causality? (There might be theoretical grounds to expect an opposite influence, that is, from uncertainty towards inflation.)
- What kind of loss for the economy is caused by price uncertainty and what is the size of this loss?

We are primarily concerned here with the second question. Our task is to check whether there is actually a connection between the rate of inflation and price uncertainty and to acquire quantitative estimates of this connection. Quantitative estimates of the loss caused by price uncertainty are outside the considerations of this paper, while the question of the direction of causality is referred to only briefly. The variables obtained from econometric models (conditional variance from GARCH-type models) are used as indicators of price uncertainty.

Let us now turn to a brief review of the empirical studies analysing the link between inflation and price uncertainty. Studies using cross-country data usually confirm the existence of this relationship (Okun, 1971; Logue and Willett, 1976; Foster, 1978; Ball and Cecchetti, 1990).

As for studies using time series data that employ GARCH-type models, the evidence is mixed. For example, in his pioneering paper, Engle (1983) was not able to find a significant association between the rate of inflation and price uncertainty in the USA. However, a number of studies in which modifications of classical GARCH regression were used have confirmed this regularity. Baillie *et al*. (1996) have found a significant dependence in a number of high inflation countries. Evans (1991) has found a significant dependence using USA data. In the paper by Pagan, Hall and Trivedi (1983), a somewhat different approach was used which did not use a GARCH specification, and this also produced results confirming the existence of the regularity.

Besides the definition of uncertainty as a conditional variance, as in GARCH analysis, there is a definition of it as a disagreement in expectations concerning the price level (see, for example, Wachtel, 1977; Pagan, Hall and Trivedi, 1983; Holland, 1995). Survey data are used in this case to estimate the dependence between uncertainty and the rate of inflation. Based on survey forecasts, indicators are built which describe how much people differ in their expectations with respect to future inflation; these indicators are taken as a measure of uncertainty and their relationship with the rate of inflation is studied.

More detailed surveys of the literature on this subject could be found in Driffill *et al*. (1990), Golob (1993) and O'Reilly (1998).

The connection between inflation and price uncertainty remains an empirical fact, although this has not been confirmed definitely by the use of time series in an individual country. It is not quite clear what may be the formal theoretical grounds for such a link.

One explanation was offered by Holland (1993). In his model, the rate of inflation and price uncertainty are connected through uncertainty in forecasts concerning the influence of money growth on the price level. This uncertainty of the influence of money growth is modelled by random coefficients of the forecast equation.

A somewhat different model was offered by Ball (1992). According to this, the higher is the average rate of inflation, the less definite are the expectations of economic agents concerning the future policy of the monetary authorities. The reason is that, when the rate of inflation is low, there exists a broad consensus among the public as to what kind of monetary policy should be followed: everyone agrees that inflation must be kept at a low level. If the rate of inflation is high, there exists political pressure in the direction of disinflation as well as in the direction of continuing inflation as a consequence of the fear of the negative effects of disinflation. Which policy will triumph is not clear.

Note that this explanation emphasises the long-run aspects of price uncertainty. However, as the current study demonstrates, the average rate of inflation is significantly correlated with a short-run variability in the price level which, from the long-run point of view, is nothing more than noise (*cf*. Ball and Cecchetti, 1990).

An explanation similar to Ball's reasoning could be found in the fundamental work on high inflation by Leijonhufvud and Heymann (1994). This is based on a concept of the monetary policy regime. High inflation is usually accompanied by adaptive actions by the government without firm rules; decisions are made under the influence of current needs.

2. THE RELATIONSHIP BETWEEN INFLATION AND PRICE UNCERTAINTY: THE RUSSIAN EXPERIENCE

The main problem which one has to deal with when investigating the relationship between inflation and price uncertainty is that the latter can not be measured directly. Population surveys including forecasts of inflation are not developed in Russia, so it is necessary to use indirect estimates of uncertainty constructed on the basis of price level predictions taken from an econometric model.

It is evident that measures of price level uncertainty must somehow comprise forecast error. Let us assume that forecasts are made on the basis of the past behaviour of observable indicators (backward-looking expectations). Then, the equation for forecast error has the form:

$$\varepsilon_t = \ln P_t - f(\Omega_{t-1}),$$

where P_t is the price index, $f(.)$ is a predictor function, and Ω_{t-1} is the information available at the moment of making the forecast. We assume that systematic forecast error is almost absent, that is, expectation of ε_t conditional on Ω_{t-1} is almost zero.

The easiest and most natural way to measure uncertainty is to use the variance of forecast error (in general, this must be conditional on past information):

$$\sigma_t^2 = \mathrm{Var}(\varepsilon_t \mid \Omega_{t-1}).$$

The first candidate variable to use when making forecasts is the price level itself. It is also reasonable to suppose that the past behaviour of money (M_t) can also be important information. This reasoning results in the following equation for the modelling of forecasts of the price level:

$$f(\Omega_{t-1}) = m + \sum_{j=1}^{p} a_j \ln P_{t-j} + \sum_{j=1}^{q} b_j \ln M_{t-j} . \qquad (1)$$

Thus, we use a regression model of the following form:

$$\ln P_t = m + \sum_{j=1}^{p} a_j \ln P_{t-j} + \sum_{j=1}^{q} b_j \ln M_{t-j} + \varepsilon_t . \qquad (2)$$

This model, presumably, has a heteroskedastic error, *i.e.* its variance, σ_t^2, is not the same for different observations.

In this section, we obtain estimates of uncertainty (σ_t^2) behaviour in the Russian economy and study the link between σ_t^2 and inflation rates. Obviously, it is reasonable to analyse this relationship only in the period after price liberalisation, *i.e.* starting from 1992.

Monthly data on the consumer price index and the monetary base in the Russian economy from 1992 to the beginning of 2000 were used in computations. The source of data was *Russian Economic Trends*.[2] This source gives a sufficiently complete set of the necessary macroeco-

[2] The data are available on the Internet at: http: //www.hhs.se/site/ret/ret.htm.

nomic data; the published series do not contain gaps and the period of time covered by them is fairly large.

Before turning to models of the relationship between inflation and price uncertainty, we obtain preliminary estimates of price uncertainty behaviour based on regression (2). We use a representation of the logarithm of variance by polynomial trend as a means of modelling changes in variance, that is:

$$\ln \sigma_t^2 = \sum_{k=0}^{K} \alpha_k t^k . \tag{3}$$

From the point of view of econometric modelling, equations (2) and (3) define a regression model with multiplicative heteroskedasticity.

Tables 1 and 2 show the estimates of regression (1), in which both lag lengths (p and q) were set to be 4 months and the order of the polynomial was chosen to be 9.[3] When estimating equation (1), the consumer price index was used as P_t and the monetary base as M_t.

Table 1.

$\ln P_t =$	0.1778	$+ 1.982 \ln P_{t-1}$	$- 1.133 \ln P_{t-2}$	$+ 0.144 \ln P_{t-3}$	$+ 0.021 \ln P_{t-4}$
	(1.67)	(19.0)	(−5.12)	(0.70)	(0.22)
	[0.100]	[0.000]	[0.000]	[0.484]	[0.827]
	$+ 0.047 \ln M_{t-1}$	$- 0.086 \ln M_{t-2}$	$+ 0.034 \ln M_{t-3}$	$- 0.016 \ln M_{t-4}$	
	(4.47)	(−6.7)	(1.29)	(−0.73)	
	[0.000]	[0.000]	[0.200]	[0.465]	
	95 observations (May 1992 − March 2000)				

Table 2. Multiplicative heteroskedasticity parameters.

$\ln \sigma_t^2 =$	-18.73	$+ 6.677\, t$	$- 1.165\, t^2$	$+ 0.0945\, t^3$	$- 0.00421\, t^4$	$+ 1.118\times10^{-4}\, t^5$
	(−1.68)	(1.70)	(−2.19)	(2.58)	(−2.88)	(3.13)
	[0.010]	[0.094]	[0.031]	[0.012]	[0.005]	[0.003]
	$-1.808\times10^{-6}\, t^6$	$+ 1.749\times10^{-8}\, t^7$	$- 9.294\times10^{-11}\, t^8$	$+ 2.083\times10^{-13}\, t^9$		
	(−3.35)	(3.56)	(−3.76)	(3.94)		
	[0.001]	[0.001]	[0.000]	[0.000]		

[3] The notation used in the tables is explained in the Appendices.

Fig. 1 shows the behaviour of variance calculated according to the obtained estimates.[4] A sharp upward jump in variance can be seen on the chart, which corresponds to August and Autumn 1998.

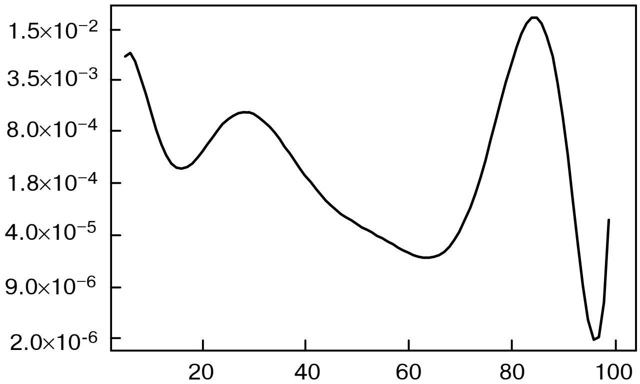

Figure 1. Variance estimated by polynomial trend.

It could be conjectured that, in August 1998, as a result of the crises, there was a structural break which caused a shift in variance. Thus, one gets the idea to estimate a model with different polynomials for the period preceding August 1998 and for that after August 1998. The estimates are shown in Tables 3 and 4. For the first, longer period, a 5th degree polynomial was chosen. For the second period a 3rd degree polynomial was chosen.

The behaviour of variance based on the estimates is shown in Fig. 2.

Of course, these estimates of variance could not provide sound estimates of forecast uncertainty, because they are based on an assumption of a deterministic "smooth" behaviour of heteroskedasticity. These should be considered as tentative preliminary estimates.

A more appropriate way of modelling changes in variance is the so-called GARCH process, *i.e*. a process with autoregressive conditional heteroskedasticity. If ε_t is a GARCH(k, l) process, then it is defined by the following recursion for the variance of the process:

$$\sigma_t^2 = \mu + \sum_{j=1}^{k} \gamma_j \varepsilon_{t-j}^2 + \sum_{j=1}^{l} \delta_j \sigma_{t-j}^2 , \qquad (4)$$

[4] Both here and below, the first observation corresponds to January 1992.

Table 3. Multiplicative heteroskedasticity parameters (before August 1998).

$$\ln \sigma_t^2 = 4.785 - 2.344\,t + 0.1567\,t^2 - 0.0046\,t^3 + 5.932\times10^{-5}\,t^4 - 2.813\times10^{-7}\,t^5$$

(1.90)	(−4.75)	(4.84)	(−4.95)	(4.98)	(−4.99)
[0.061]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

Table 4. Multiplicative heteroskedasticity parameters (after August 1998).

$$\ln \sigma_t^2 = 3667.1 - 118.49\,t + 1.272\,t^2 - 0.004551\,t^3$$

(2.47)	(−2.37)	(2.28)	(−2.19)
[0.016]	[0.020]	[0.025]	[0.032]

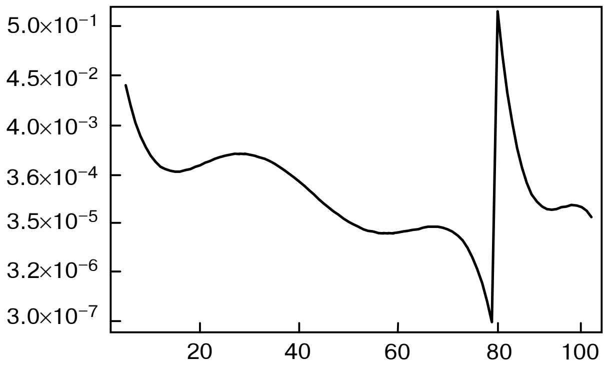

Figure 2. Variance estimated by polynomial trend with break.

where

$$E(\varepsilon_t \mid \Omega_{t-1}) = 0,$$

and

$$\mathrm{Var}(\varepsilon_t \mid \Omega_{t-1}) = \sigma_t^2 .$$

The models of the GARCH type are widely utilised in modelling the uncertainty (volatility) of financial time series and price indices.

We used a GARCH(1,1) model for the error in regression (2). There is no point in taking $k > 1$ or $l > 1$, because estimates of the corresponding coefficients prove to be insignificant. To allow for the "August crises" of 1998, August 1998 and September 1998 dummies were added to the equation. Tables 5 and 6 show the estimates for the model.

Table 5. Regression with GARCH(1,1) error.

$\ln P_t =$	0.1853	$+ 1.802 \ln P_{t-1}$	$- 1.033 \ln P_{t-2}$	$+ 0.386 \ln P_{t-3}$	$- 0.149 \ln P_{t-4}$
	(2.97)	(20.7)	(−5.77)	(2.57)	(−2.53)
	[0.004]	[0.000]	[0.000]	[0.012]	[0.013]
	$+ 0.011 \ln M_{t-1}$	$- 0.055 \ln M_{t-2}$	$+ 0.046 \ln M_{t-3}$	$- 0.021 \ln M_{t-4}$	
	(0.76)	(−2.91)	(2.57)	(−1.63)	
	[0.449]	[0.005]	[0.012]	[0.108]	
	95 observations (May 1992 – March 2000)				

Table 6. Parameters of GARCH(1,1) process.

Parameter	Estimate	t-stat.	[sign.]
μ	6.116×10^{-6}	1.41	[0.162]
γ_1	1.137	4.07	[0.000]
δ_1	0.162	3.27	[0.000]

As is well known, when:

$$\sum_{j=1}^{k} \gamma_j + \sum_{j=1}^{l} \delta_j \geq 1,$$

the GARCH process (4) is non-stationary.[5] If the inequality were strict, this would be an "explosive" process. For our model:

$$\gamma_1 + \delta_1 = 1.137 + 0.162 = 1.299.$$

So, the estimates are within the nonstationarity region.

Fig. 3 shows a plot of conditional variance calculated from the model estimates. Its behaviour is very similar to that of the estimates of variance derived from the polynomial trend model (see Figs 1 and 2).

[5] The GARCH model can be rewritten as an ARMA model for squared residuals. In order for the process to be stationary, the roots of the corresponding characteristic polynomial must be outside the unit circle. If the given condition is true then one of the roots lies on the unit circle or inside it, *i.e*. the process is nonstationary.

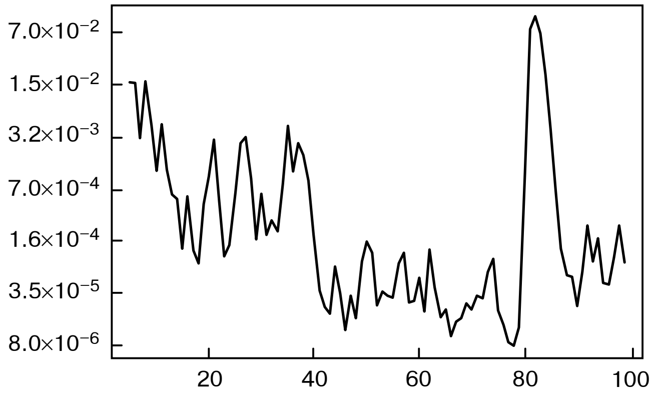

Figure 3. Conditional variance of the GARCH process.

The two estimates of variance behaviour — based on the trend including a break and on GARCH — are shown in Fig. 4. As can be seen from the plot, the main point of difference between the graphs is at Summer and Autumn 1998. This is caused most probably by the end effects in the

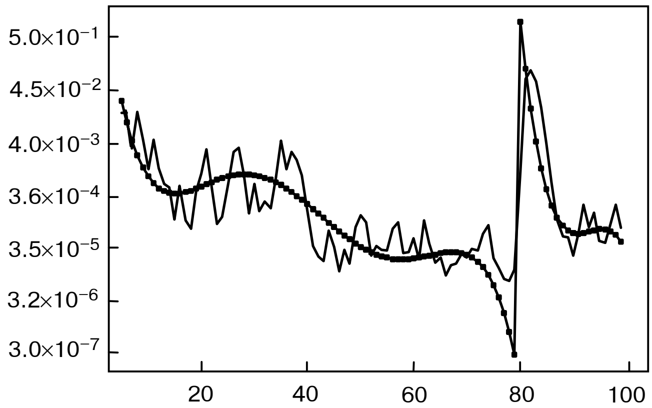

Figure 4. Two variance estimates compared.

estimate being obtained from the model with a trend polynomial.
An analysis of the plots helps to elicit several periods from the point of view of variance behaviour:

1) (Later half of 1992) A decline in variance, starting from a higher level induced by price liberalisation and the beginning of market reforms.

2) (1993 – beginning of 1995) Stable and high level of variance.
3) (Spring 1995 – Summer 1998) Lower level of variance.
4) (Autumn of 1998) Variance leaps upwards as a result of crises.
5) (1999 – beginning of 2000) Variance lowered to a level higher than it was before the crises.

Let us remember that our basic hypothesis states that the higher is inflation, the higher would be the uncertainty. Thus, it is necessary to include the inflation rate ($\pi_t = \Delta \ln P_t$) in the model. For the GARCH(k, l) process, this relationship takes the following form:

$$\sigma_t^2 = \mu + \sum_{j=1}^{k} \gamma_j \varepsilon_{t-j}^2 + \sum_{j=1}^{l} \delta_j \sigma_{t-j}^2 + \lambda \pi_{t-1}. \tag{5}$$

If one fails to reject the null hypothesis:

H_0: $\lambda = 0$,

then there is evidence of a hypothesised dependence between uncertainty and the rate of inflation. The expectation is that $\lambda > 0$.

Regretfully, we were not able to obtain estimates for this model because the algorithm utilised failed to converge.

We may still compare the behaviour of the conditional estimates of variance, which were previously obtained for GARCH without allowing for inflation rate, to the behaviour of the rate of inflation.

Fig. 5 shows the behaviour of the logarithm of conditional variance and of inflation (the conditional variance was scaled so that the plots became comparable). As is seen from the diagram, the trends are roughly the same but the short-run movements are substantially different.

Note that an important point here is that it is the *logarithm* of conditional variance which we are comparing with inflation. Fig. 6 shows the behaviour of the inflation rate together with conditional variance (without taking logarithms). It is evident that, in this form, the two indicators are absolutely incomparable with each other. Thus, the problems with convergence might be due to an inappropriate choice of the functional form of the relationship.

Taking this into consideration, we have modified the model by turning to logarithms in a GARCH process:

$$\ln \sigma_t^2 = \mu + \sum_{j=1}^{k} \gamma_j \frac{\varepsilon_{t-j}^2}{\sigma_{t-j}^2} + \sum_{j=1}^{l} \delta_j \ln \sigma_{t-j}^2 . \tag{6}$$

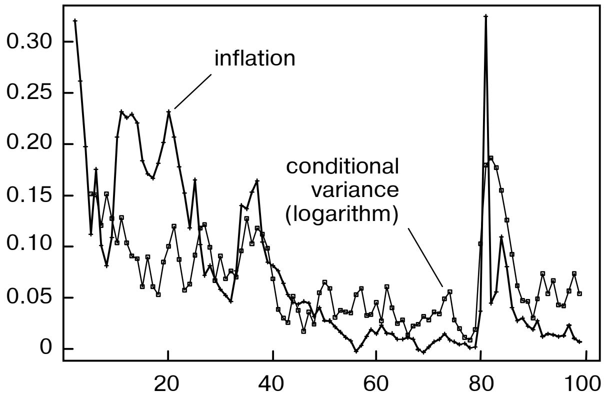

Figure 5. Logarithm of conditional variance compared with the inflation rate.

This model (it could be termed a logarithmic GARCH model) is somewhere between the classical GARCH model (4) and the "exponential" GARCH model.[6]

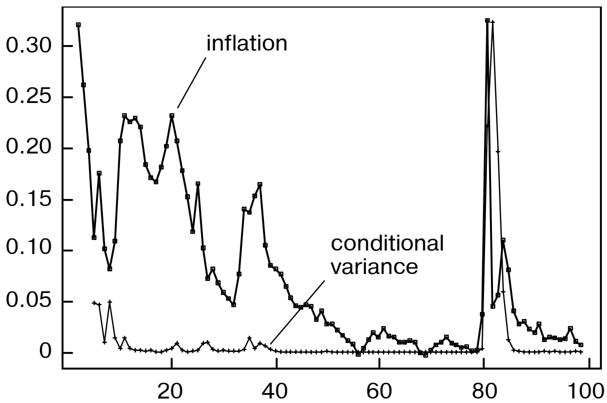

Figure 6. Conditional variance compared with inflation rate.

[6] Model (6) could be considered as a first-order approximation to model (4) if one takes $\mu = 0$ in (4). Taking this into account, it could be expected that the estimates of the conditional variance of the two models do not differ greatly, as the estimate of μ in Table 6 is insignificant.

As before, we estimated model (6) with $k = 1$ and $l = 1$, adding to the GARCH process two dummy variables corresponding to the crisis. The estimates of the coefficients of the logarithmic GARCH process are shown in Table 7.

Table 7. Parameters of logarithmic GARCH(1,1) process.

Parameter	Estimate	t-stat.	[sign.]
μ	−1.869	−5.65	[0.000]
γ_1	0.881	5.701	[0.000]
δ_1	0.908	22.27	[0.000]

Two estimates of conditional variance, from classical and logarithmic GARCH models, happen to be very close to each other (see Fig. 7).

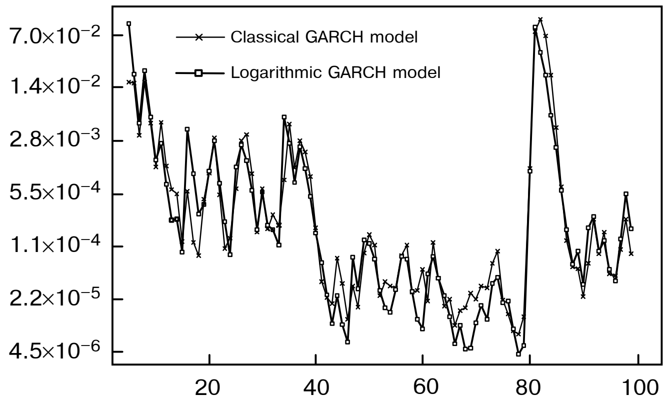

Figure 7. Conditional variance estimates from the two models compared.

Let us now check whether conditional variance depends on lagged inflation in this modified model. Namely, in a regression with conditional variance given by the equation:

$$\ln\sigma_t^2 = \mu + \sum_{j=1}^{k}\gamma_j \frac{\varepsilon_{t-j}^2}{\sigma_{t-j}^2} + \sum_{j=1}^{l}\delta_j \ln\sigma_{t-j}^2 + \lambda\,\pi_{t-1} \qquad (7)$$

we can test the hypothesis:

H_0: $\lambda = 0$.

Coefficient λ could be termed the semi-elasticity of conditional variance with respect to the inflation rate. Estimates of the model are shown in Table 8.

Table 8. Parameters of the logarithmic GARCH(1,1) process: the relationship between the logarithm of conditional variance and the inflation rate.

Parameter	Estimate	t-stat.	[sign.]
μ	−6.421	−5.26	[0.000]
γ_1	0.454	3.331	[0.001]
δ_1	0.415	3.54	[0.001]
λ	14.4	3.95	[0.000]

As can be seen from the table, λ proved to be significant. According to this estimate, a 1 percentage point change in the inflation rate is accompanied by a 14.4 per cent change in conditional variance (in a logarithmic ratio).

These estimates were obtained without adding dummy variables for August and September 1998 to the conditional variance equation (7). If these variables are added, the situation changes dramatically. Corresponding estimates are presented in Table 9.

Table 9.

Parameter	Estimate	t-stat.	[sign.]
μ	−2.510	−5.33	[0.000]
γ_1	0.848	5.77	[0.000]
δ_1	0.851	17.7	[0.001]
λ	2.496	1.60	[0.113]

The estimate of λ is of the expected sign, but it is not significant at the 10 per cent level. Thus, one can not be sure that the results in Table 8 did not appear by coincidence.

Testing the null hypothesis $\lambda = 0$ in this way can be considered as a test of the absence of Granger causality going from inflation to price uncertainty. Acceptance of the hypothesis $\lambda = 0$ can be interpreted as follows: lagged inflation does not contain any useful information for predicting conditional variance beyond the information which is contained in other relevant variables.

There exist theoretical explanations of the link between inflation and price uncertainty in which causality goes from uncertainty to inflation. The corresponding hypothesis can be tested with the help of a model of the GARCH-M (GARCH in mean) type. The model is constructed by augmenting equation (2) with a dependence on the logarithm of conditional variance:

$$\ln P_t = m + \sum_{j=1}^{p} a_j \ln P_{t-j} + \sum_{j=1}^{q} b_j \ln M_{t-j} + \rho \ln\sigma_t^2 + \varepsilon_t. \quad (8)$$

This causal interpretation of a GARCH-M model implies that the conditional variance, σ_t^2, is known before the price level, P_t, becomes known.

The estimates of model (8) with an error variance driven by a logarithmic GARCH(1,1) process (7) are shown in Tables 10 and 11.

Table 10. Logarithmic GARCH-M(1,1) regression.

$\ln P_t =$	0.2495	$+ 1.801 \ln P_{t-1}$	$- 0.990 \ln P_{t-2}$	$+ 0.237 \ln P_{t-3}$	$- 0.039 \ln P_{t-4}$
	(7.70)	(23.6)	(−6.15)	(2.01)	(−1.05)
	[0.000]	[0.000]	[0.000]	[0.048]	[0.296]
	$- 0.013 \ln M_{t-1}$	$- 0.049 \ln M_{t-2}$	$+ 0.063 \ln M_{t-3}$	$- 0.026 \ln M_{t-4}$	$+ 0.2395 \ln\sigma_t^2$
	(−1.32)	(−4.29)	(5.54)	(−3.52)	(2.84)
	[0.190]	[0.000]	[0.000]	[0.001]	[0.006]
95 observations (May 1992 − March 2000)					

Table 11. Parameters of a logarithmic GARCH(1,1) process for GARCH-M regression.

Parameter	Estimate	t-stat.	[sign.]
μ	−1.706	−5.69	[0.000]
γ_1	0.866	5.89	[0.000]
δ_1	0.928	24.5	[0.000]

Coefficient ρ proved to be positive and significant. In the spirit of the Granger causality concept, this result could be interpreted as follows:

uncertainty can causally influence inflation, inducing growth in the price level.

Following the same logic, models (7) and (8) could be merged.

The estimates of model (8) with an error variance modelled by a logarithmic GARCH(1,1) process (6) are shown in Tables 12 and 13.

Table 12. Logarithmic GARCH-M(1,1) regression.

$\ln P_t =$	0.2329	$+ 1.696 \ln P_{t-1}$	$- 0.901 \ln P_{t-2}$	$+ 0.366 \ln P_{t-3}$	$- 0.150 \ln P_{t-4}$
	(5.87)	(27.4)	(−6.95)	(3.37)	(−3.74)
	[0.000]	[0.000]	[0.000]	[0.001]	[0.000]
	$+ 0.007 \ln M_{t-1}$	$- 0.051 \ln M_{t-2}$	$+ 0.045 \ln M_{t-3}$	$- 0.024 \ln M_{t-4}$	$+ 0.5858 \ln \sigma_t^2$
	(0.89)	(−6.26)	(4.29)	(−3.64)	(6.19)
	[0.375]	[0.000]	[0.000]	[0.001]	[0.006]
95 observations (May 1992 − March 2000)					

Table 13. Parameters of a logarithmic GARCH(1,1) process for GARCH-M regression.

Parameter	Estimate	t-stat.	[sign.]
μ	−4.260	−6.84	[0.000]
γ_1	0.870	5.91	[0.000]
δ_1	0.712	11.7	[0.000]
λ	9.091	5.52	[0.000]

Both coefficient ρ and coefficient λ are significant in this model and have the expected signs.

The obtained estimates should be interpreted as an evidence of a bilateral relationship between the rate of inflation and price uncertainty, taking the form of a spiral:

$$\textit{inflation} \rightarrow \textit{uncertainty} \rightarrow \textit{inflation}.$$

Hence, based on the Russian data, a confirmation of a link between the rate of inflation and uncertainty is obtained. But this evidence is not uncontroversial because the estimates in Tables 12 and 13 were computed using only 95 observations.[7]

[7] 95 observations were used to estimate 17 parameters, including two "crisis" dummies and the value of the logarithm of conditional variance at moment zero (they are not shown in the table in order to save space).

3. THE RELATIONSHIP BETWEEN INFLATION AND PRICE UNCERTAINTY: COMPARISON WITH OTHER COUNTRIES

In this section we try to find out whether the Russian experience is in accord with international experience. We use the following basic model to study the link between inflation and price uncertainty on the basis of cross-country data:

$$\sigma_i = \alpha + \beta\ \bar{\pi}_i + u_i, \qquad (9)$$

where σ_i is some measure of uncertainty for the i-th country, $\bar{\pi}_i$ is the average inflation rate, and u_i is random error. As was mentioned in the Introduction, studies of this kind have already been carried out several times (although the methods used have been somewhat different). Within the framework of the project, the purpose of exploring this link is to test statistically the hypothesis that the Russian experience is in line with the general tendency.

The simplest course is to use the standard deviation (SD) of the inflation rate for some period of length T as a measure of price variability:

$$SD = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\pi_t - \bar{\pi})^2}\ ,$$

where:

$$\pi_t = \Delta \ln P_t,$$

$$\bar{\pi} = \frac{1}{T}\sum_{t=1}^{T}\pi_t = \frac{\ln P_T - \ln P_0}{T}\ .$$

The data analysed are the price indices for a set of different countries and different periods. Each observation is based upon monthly data for two successive years. To decrease the autocorrelation in observations from the same country, the periods were chosen in such a way that they are separated by four-year intervals. Some data are from periods of high inflation (including those analysed in the well-known papers by Cagan (1956) and Sargent (1982). But most of the data can be characterised as from periods of moderate inflation. The sources are listed in the Appendix. The data are from 128 observations (Russia included).

In the estimated equation, the standard deviation of the rate of inflation (SD_i) was the dependent variable and the average inflation rate $(\bar{\pi}_i)$ was the explanatory variable. The results are shown in Table 14.

Table 14.

$SD_i = 0.1813 + 0.7797\ \bar{\pi}_i$
(1.181) (22.7)
[0.240] [0.000]
128 observations, $R^2 = 80.191\%$
$F(1,126) = 515.13$ [0.000]
Normality: $\chi^2(2)= 744.6$ [0.000]
Heteroskedasticity: $\chi^2(1) = 213.6$ [0.000]
Functional form: $\chi^2(1) = 32.01$ [0.000]

The t-statistic for the coefficient of $\bar{\pi}_i$ shows a significant positive relationship. But the estimated regression demonstrates poor diagnostic statistics, revealing the presence of a statistically-significant specification error. As the histogram shows (see Fig. 8), the distribution of errors possesses strong positive excess kurtosis (for the purpose of comparison, normal density is also shown on the histogram). The non-normality of the error term here may result from heteroskedasticity because, evidently from Fig. 9, the residuals are more dispersed at the higher levels of inflation.

To cope with the revealed specification errors, a variance-stabilising transformation was used. Standard deviation, SD_i, was replaced by its

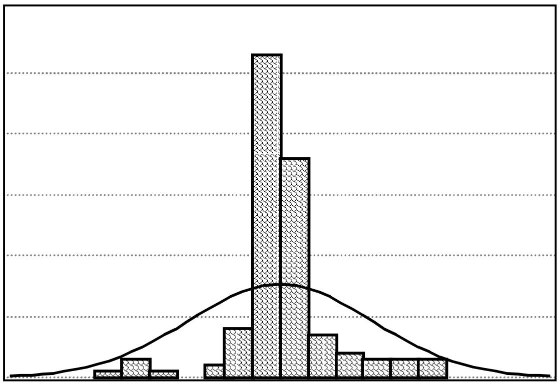

Figure 8. Histogram of residuals from the regression of the standard deviation of inflation on the average rate of inflation.

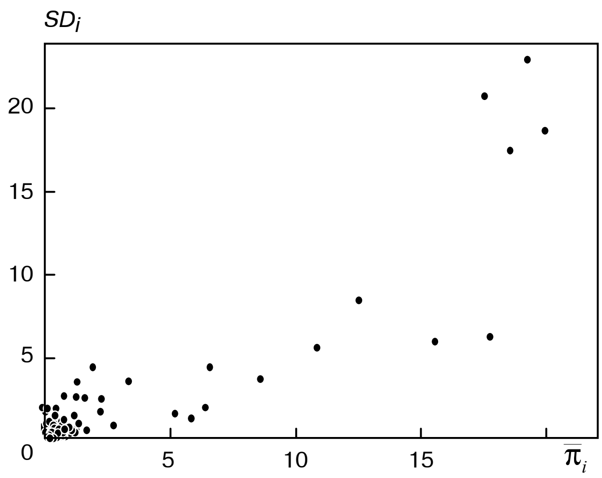

Figure 9. Regression of the standard deviation of inflation on average inflation.

logarithm, ln SD_i, in the role of dependent variable. This transformation is feasible because standard deviation is always positive. The resulting model is:

$$\ln SD_i = \alpha + \beta \bar{\pi}_i + u_i. \quad (10)$$

Estimation results are presented in Table 15.

Table 15.

ln $SD_i = -0.7367 + 0.2012\ \bar{\pi}_i$ (−11.3) (13.8) [0.000] [0.000]
128 observations, R^2 = 59.843% $F(1,126)$ = 190.26 [0.000]
Normality: $\chi^2(2)$ = 7.57 [0.022] Heteroskedasticity: $\chi^2(1)$ = 0.70 [0.402] Functional form: $\chi^2(1)$ = 9.50 [0.002]

As can be seen from the table, the diagnostics become dramatically better. The lower coefficient of determination is not a problem because it is impossible to compare two models with different dependent variables on the basis of this indicator.

However, in this case functional form mis-specification becomes evident. We may conjecture that the relation is non-linear. The easiest way to model non-linearity is by polynomial in $\bar{\pi}_i$:

$$\ln SD_i = \sum_{k=0}^{K} \beta_k (\bar{\pi}_i)^k + u_i. \quad (11)$$

Experiments with different K suggest a 3rd degree polynomial (K = 3, Table 16).

Table 16.

$\ln SD_i = -0.9837 + 0.6655\ \bar{\pi}_i - 0.0598\ \bar{\pi}_i^2 + 0.00186\ \bar{\pi}_i^3$				
	(−11.5)	(5.68)	(−3.32)	(2.78)
	[0.000]	[0.000]	[0.001]	[0.006]
128 observations, R^2 = 64.445%				
$F(3,124)$ = 77.732 [0.000]				
Normality: $\chi^2(2)$ = 9.10 [0.028]				
Heteroskedasticity: $\chi^2(1)$ = 1.14 [0.285]				
Functional form: $\chi^2(1)$ = 2.08 [0.149]				

Of the three diagnostic statistics, only the one for normality is significant at the 10% level, although the departure from normality is not particularly large (see Fig. 10).

However, in this sample Russia is represented by 3 observations which belong to the periods July 1992–June 1994, July 1994–June 1996 and July 1996–June 1998. All three observations lie fairly closely to the estimated curve (see Fig. 11). We should formally test the hypothesis that the observations on Russian inflation belong to the same sample as the other observations.

Suppose that i_0th observation in the sample correspond to Russia. We can construct the following dummy variable:

$D_i = 1$, if $i = i_0$,

and

$D_i = 0$, if $i \neq i_0$.

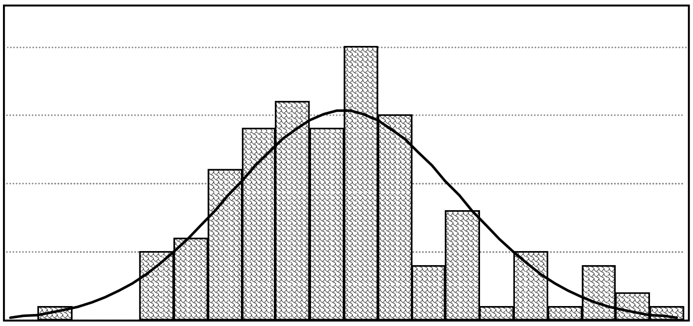

Figure 10. Histogram of residuals from the regression of the logarithm of the standard deviation of inflation on the average inflation rate

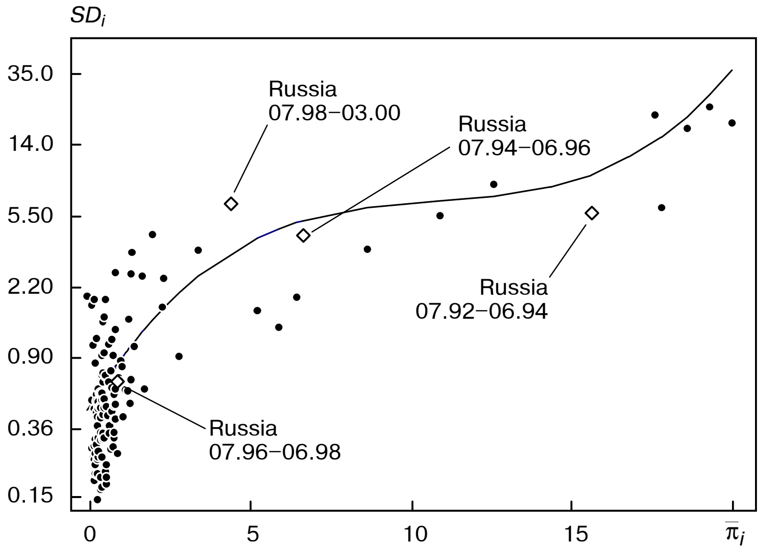

Figure 11. The relationship between the standard deviation of inflation (logarithmic scale) and inflation.

Test the hypothesis that observation i_0 is from the same model as other observations is the same as to test the hypothesis:

H_0: $\gamma = 0$

in the regression:

$$\ln SD_i = \alpha + \sum_{k=0}^{K} \beta_k (\bar{\pi}_i)^k + \gamma D_i + u_i.$$

If the null hypothesis cannot be rejected, then one has to conclude that the experience of Russia fits well with the world experience.

Three regressions were estimated, in all of which a dummy variable was added corresponding to one of the three periods. A t-statistic (possessing 123 degrees of freedom) for the hypothesis that an observation is from the same sample ($\gamma = 0$) was calculated in each regression.

The statistics and their significance levels are shown in Table 17.

Table 17.

Period	t-statistics	Significance level
July 1992 – June 1994	−0.252	[0.801]
July 1994 – June 1996	0.260	[0.795]
July 1996 – June 1998	0.131	[0.896]

All three statistics are insignificant, so in all three cases we have to accept the hypothesis that the coefficient γ is zero.

A statistic for the addition of *all* three dummies can also be calculated. It is 0.047627 and distributed as Fisher F with (3, 121) degrees of freedom. The significance level is 0.9862, so the hypothesis could not be rejected.

Additional testing was made for the period July 1998–March 2000. This period is shorter than the others (21 months), so the corresponding observation was not included in the sample from which estimates were obtained. Student's statistics for the corresponding hypothesis is 1.457 and significance level is 0.1477, *i.e*. the dummy is insignificant at the 10% level.

As can be seen from Fig. 11, at the beginning of the period Russia drifted from a higher to a lower inflation rate along a gently sloping part of the curve so that the decline in price level variability was not as great as it could have been with a higher or a lower inflation rate. Then, the path followed a steeper part of the curve which, under a considerable fall in inflation, resulted in a considerable decline in uncertainty. In the period July 1998–March 2000, both inflation and uncertainty become higher, corresponding to an upward movement along the same steep part of the curve.

This analysis was based on standard deviation, which is quite a naive indicator characterising variability rather than uncertainty. As was mentioned above, it would be more adequate to measure uncertainty by forecast variance.

One such indicator that we have used is the standard error of the two months ahead forecast, which is based on an autoregressive model. This corresponds to the following predictor function:

$$f\,(\Omega_{t-1}) = \beta_0 + \beta_2 \ln P_{t-2} + \beta_3 \ln P_{t-3}\,. \qquad (12)$$

The estimation results from the regression of the logarithm of standard error from regression (12) on a third degree polynomial of the inflation rate are shown in Table 18.

Table 18.

$\ln \sigma_i = -0.7201 + 0.6778\ \bar{\pi}_i - 0.0603\ \bar{\pi}_i^2 + 0.00186\ \bar{\pi}_i^3$
(−8.77) (6.00) (−3.46) (2.88)
[0.000] [0.000] [0.001] [0.005]
128 observations, R^2 = 67.042%
F(3,124) = 87.11313 [0.000]
Normality: $\chi^2(2)$ = 4.87 [0.087]
Heteroskedasticity: $\chi^2(1)$ = 0.33 [0.566]
Functional form: $\chi^2(1)$ = 3.38 [0.066]

As can be seen from the table, these estimates closely resemble those obtained for standard deviation. The correlation between these two dependent variables is 0.9858; thus the similarity of the results is quite explicable. That so simple an indicator as standard deviation is intimately correlated with more sophisticated indicators based on complicated forecasting schemes is not contrary to the experience of previous research.

Yet another approach proceeds from an autoregressive predictor function which predicts one month ahead:

$$f\,(\Omega_{t-1}) = m + \sum_{j=1}^{p} a_j \ln P_{t-j}\,,$$

and GARCH specification for the error conditional variance:

$$\sigma_t^2 = \mu + \gamma\varepsilon_t^2 + \delta\sigma_{t-1}^2 .$$

For a typical country (or "observation"), *unconditional* variance (expected conditional variance):

$$\sigma^2 = \frac{\mu}{1 - \gamma - \delta}$$

would be a measure of overall uncertainty. Unconditional variance exists only if the GARCH process is stationary, *i.e*. if $\gamma + \delta < 1$.

In most cases, either the GARCH effect proved to be insignificant or the used algorithm failed to converge. In several cases, the estimates correspond to a non-stationary process. So, we have to give up the idea of estimating a regression similar to those above for this method.

Let us note that, from the point of view of rational expectations, all three measures are imperfect because they do not use all the information available to economic agents.

4. FINAL COMMENTS

We can draw the conclusion that cross-country data indicate that there does exist a link between inflation and price uncertainty. Dynamic models estimated on the Russian data also confirm the existence of this link. However, in this case it is impossible to draw unambiguous conclusions.

Our belief is that differences in the results obtained using cross-country and time series data can be explained by reference to the notion of a monetary policy regime which was mentioned in the Introduction. In the same country, changes in the monetary policy regime are infrequent, but the average rate of inflation and price uncertainty may vary significantly under the same regime. In this case, the link between inflation and price uncertainty is a long-run one.

At the same time, uncertainty by itself, within the framework of this link, can be short-run, as our study shows. We calculated measures of uncertainty on the basis of monthly data, but the link was apparent on time intervals of several years.

The study of the Russian data has shown that the link between inflation and price uncertainty can be bilateral. Let us note, however, that it is necessary to pay attention to the limitations peculiar to the concept of Granger causality which we used. In particular, it could be the case that both inflation and price uncertainty are determined by some third factor. For example, the monetary policy regime which we mentioned above could be one such factor. Along these lines, the following interpretation of the events of the 1998 "August crisis" is possible: the crisis gave rise both to anticipations of future price increases and a greater uncertainty in expectations; both of these caused a decrease in the demand for money which resulted in a growth in the price level. Thus, the results which we obtained cannot be given a definitive causal interpretation.

One of the results of this study is related to the functional form of the postulated relationship between inflation and price uncertainty. During the construction of the models, both for the cross-country data and for the Russian data, we found it more adequate to use a logarithm of the variance in the equations. The result may be technical, but its importance should not be underestimated. If our conclusion is correct and this specification is more suitable, then it gives ground to question the estimates obtained in the previous empirical studies and the conclusions made there, because the studies with which we are familiar do not use the logarithm of variance.

Hence, our analysis gives one more confirmation to a positive relationship between inflation and price uncertainty on a cross-country basis, which a number of researchers have pointed to on more than one occasion. The difference from the previous research is that we have used monthly series and/or a more adequate functional form (that is, a logarithmic transformation of a measure of variance). The main conclusion is that the Russian experience in this respect fits well with the world experience.

APPENDICES

A. Notation

We used the following notation in the tables with regression estimation results. Beneath the coefficients, the estimated t-statistics are placed in parentheses while their significance levels are in square brackets.

After this comes the number of observations.

R^2 is the coefficient of determination adjusted for degrees of freedom.

F is the F-statistic with degrees of freedom in parentheses.

Normality is the diagnostic statistic for the hypothesis that model errors are normally distributed. It has an asymptotic chi-square distribution with 2 degrees of freedom.

Heteroskedasticity is the diagnostic statistic for the hypothesis that model errors have the same variance. It has an asymptotic chi-square distribution with 1 degree of freedom.

Functional form is the diagnostic statistic for the hypothesis that the model has a correct functional form. It has an asymptotic chi-square distribution with 1 degree of freedom.

These three statistics are followed by their significance levels in square brackets.

B. Data sources for regressions based on inflationary episodes

Most of the data were obtained from papers analysing the monthly series of price indices in various countries: (1) Culver and Papell (1997), (2) Baillie *et al*. (1996), (3) Cagan (1956), (4) Sargent (1982). The data from the first two papers are available on the Internet in the *Journal of Applied Econometrics Data Archive* [http://qed.econ.queensu.ca/jae/]. Some data are from the *IMF International Financial Statistics*. The data on Russia are from *Russian Economic Trends*.

In most cases, the consumer price index was used. Data on the high inflationary episodes studied by Cagan and Sargent make the exceptions. The number of observations for each country is shown in Table 19.

Table 19.

Country	Number of observations	Country	Number of observations
Argentina	6	Japan	8
Belgium	7	Luxembourg	7
Brazil	6	Netherlands	7
Canada	8	Norway	7
Finland	7	Spain	7
France	8	U.K.	8
Germany	10	U.S.	9
Israel	7	Russia, 1992–1998	3
Italy	8	Other	5

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